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WBJEE (Engineering) : 2023

(Solutions)

Test Booklet Code

Date : 30.04.2023

MATHEMATICS

Category – I (Q.1 to 50)

(Carry 1 mark each. Only one option is correct. Negative marks : – ¼)

1. Solution : (B) ; $\dfrac{a_1+a_2+......+a_n}{n}$

$$\lim_{x\to\infty}\left\{x-\sqrt[n]{(x-a\ \ )(x-a_2)....(x-a_n)}\right\}$$

$$=\lim_{z\to 0}\left[\frac{1}{z}-\sqrt[n]{\left(\frac{1}{z}-a_1\right)\left(\frac{1}{z}-a_2\right)....\left(\frac{1}{z}-a_n\right)}\right]$$

$$=\lim_{z\to 0}\left[\frac{1}{z}-\frac{\{(1-a_1z)(1-a_2z).....(1-a_nz)\}^{1/n}}{z}\right]$$

$$=\lim_{z\to 0}\frac{1-\{(1-a_1z)(1-a_2z)....(1-a_nz)\}^{1/n}}{z}\left(\frac{0}{0}\text{form}\right)$$

$$=\lim_{z\to 0}-\frac{1}{n}\{(1-a_1z)(1-a_2z)....(1-a_nz)\}^{\frac{1}{n}-1}\ [-a_1(1-a_2z)\ ...\ (1-a_nz)-a_2(1-a_1z)\ ...$$

$$(1-a_nz)\ ...\ -a_n(1-a_1z)\ ...\ (1-a_{n-1}z)]$$

$$=\left(-\frac{1}{n}\right)(-a_1-a_2-.........-a_n)$$

$$=\frac{a_1+a_2+......+a_n}{n}$$

2. Solution : (D) ; f′ (1) exists and is 10

$$f(1^-)=\lim_{h\to 0}e^{(1-h)^{10}-1}+(1-h-1)^2\sin\frac{1}{1-h-1}$$

$$=e^0+0\times\sin\left(-\frac{1}{h}\right)=1$$

$$f(1^+) = \lim_{h\to 0} e^{(1+h)^{10}-1} + (1+h-1)^2 \sin\frac{1}{1+h-1}$$

$$= e^0 + 0 \times \sin\left(\frac{1}{h}\right)$$

$= 1$

$\therefore$ f(x) is continuous at x = 1

$$Rf'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0} \frac{e^{(1+h)^{10}-1} + (1+h-1)^2 \sin\left(\frac{1}{1+h-1}\right) - 1}{h}$$

$$= \lim_{h\to 0} \frac{e^{(1+h)^{10}-1} - 1}{h}$$

$$= \lim_{h\to 0} \frac{e^{(1+h)^{10}-1} - 1}{h} \quad \left[\frac{0}{0}\text{form}\right]$$

$$= \lim_{h\to 0} \frac{e^{(1+h)^{10}-1} . 10(1+h)^9}{1}$$

$= e^0.10.1 = 10$

$$Lf'(1) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$$

$$= \lim_{h\to 0} \frac{e^{(1-h)^{10}-1} + (1-h-1)^2 \sin\frac{1}{1-h-1} - 1}{-h}$$

$$= \lim_{h\to 0} \frac{e^{(1-h)^{10}-1} - 1}{-h} \quad \left[\frac{0}{0}\text{form}\right]$$

$$= \lim_{h\to 0} \frac{e^{(1-h)^{10}-1} . 10(1-h)^9 . (-1)}{-1} = 10$$

3. Solution : (B) f(3) – f(1) = 5 does not hold ;

$$\frac{f(3)-f(1)}{3-1} = f'(x)$$

$$\Rightarrow \frac{f(3)-f(1)}{2} = f'(x)$$

$\Rightarrow$ $f(3) - f(1) = 2f'(x) = 2\{f(x)\}^2 + 8 \geq 8$

$\therefore$ f(3) – f(1) = 5 does not hold

4. Solution : (C) equal to 3;

$$L=\lim_{h\to 0}\frac{f(2+2h+h^2)-f(2)}{f(1+h-h^2)-f(1)} \quad \left[\frac{0}{0}\text{ form}\right]$$

$$L=\lim_{h\to 0}\frac{f'(2+2h+h^2)(2+2h)}{f'(1+h-h^2)(1-2h)}$$

$$=\frac{f'(2)\cdot 2}{f'(1).1}=\frac{6\cdot 2}{4}=3$$

5. Solution : (B) $A = x^2$, $B = x$, $C = n^2$;

$$\cos^{-1}\left(\frac{y}{b}\right)=\log_e\left(\frac{x}{n}\right)^n=n\log_e\left(\frac{x}{n}\right)=n[\ell n\, n-\ell n\, n]$$

$$\Rightarrow -\frac{1}{\sqrt{1-\frac{y^2}{b^2}}}\cdot\frac{1}{b}\cdot\frac{dy}{dx}=n\left[\frac{1}{x}-0\right]$$

$$\Rightarrow -\frac{1}{\sqrt{b^2-y^2}}\frac{dy}{dx}=\frac{n}{x}$$

$$\Rightarrow x^2\left(\frac{dy}{dx}\right)^2=n^2(b^2-y^2)=n^2b^2-n^2y^2$$

$$\Rightarrow 2x\left(\frac{dy}{dx}\right)^2+x^2.2\frac{dy}{dx}.\frac{d^2y}{dx^2}=-n^2\cdot 2y\frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx}+x^2\frac{d^2y}{dx^2}=-n^2y$$

$$\Rightarrow x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+n^2y=0$$

$A = x^2$, $B = x$, $C = n^2$

6. Solution : (C) $\dfrac{-x}{\cos x(x\sin x+\cos x)}$;

$$I=\int\frac{x^2\,dx}{(x\sin x+\cos x)^2}$$

$$=\int\frac{x.x\cos x\,dx}{\cos x(x\sin x+\cos x)^2}$$

$x \sin x + \cos x = t$
$(\sin x + x \cos x - \sin x)\, dx = dt$
$x \cos x\, dx = dt$

$$=\frac{x}{\cos x}\cdot\left(-\frac{1}{x\sin x+\cos x}\right)-\int\frac{\cos x-x(-\sin x)}{\cos^2 x}\cdot\left(-\frac{1}{x\sin x+\cos x}\right)dx$$

$$=-\frac{x}{\cos x(x\sin x+\cos x)}+\int\sec^2 x\,dx$$

$$= -\frac{x}{\cos x(x\sin x+\cos x)} + \int \sec^2 x\,dx \quad = -\frac{x}{\cos x(x\sin x+\cos x)} + \tan x + c$$

$$\therefore f(x) = -\frac{x}{\cos x(x\sin x+\cos x)}$$

7. Solution : (D) 12;

$$\frac{1}{(x+1)(x-2)(x-3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$= \frac{A(x^2-5x+6)+B(x^2-2x-3)+C(x^2-x-2)}{(x+1)(x-2)(x-3)}$$

$$\left.\begin{array}{l}\left.\begin{array}{l}\therefore A+B+C=0,\\ -5A-2B-C=0\end{array}\right\}\Rightarrow -4A-B=0\\ \left.\begin{array}{l}6A-3B-2C=1\\ -10A-4B-2C=0\end{array}\right\}\Rightarrow 16A+B=1\end{array}\right\}\Rightarrow 12A=1 \Rightarrow A=\frac{1}{12}$$

$$B = -4A = -4\times\frac{1}{12} = -\frac{1}{3}$$

$$C = -A-B = -\frac{1}{12}+\frac{1}{3} = \frac{-1+4}{12} = \frac{3}{12} = \frac{1}{4}$$

$$\therefore I = \int\left(\frac{1}{12}\cdot\frac{1}{x+1} - \frac{1}{3}\cdot\frac{1}{x-2} + \frac{1}{4}\cdot\frac{1}{x-3}\right)dx$$

$$= \frac{1}{12}\cdot \ln|x+1| - \frac{1}{3}\ln|x-2| + \frac{1}{4}\ln|x-3|$$

$$= \frac{1}{12}\ln|x+1| - \frac{1}{12}\ln|x-2|^4 + \frac{1}{12}\ln|x-3|^3$$

$$= \frac{1}{12}\ln\left\{\frac{|x+1||x-3|^3}{(x-2)^4}\right\}$$

$\therefore$ k = 12

8. Solution : (D) n – 1;

$$\int_0^n [x]dx$$

$$= \int_0^1 [x]dx + \int_1^2 [x]dx + \cdots + \int_{n-1}^n [x]dx$$

$$= 0 + \int_1^2 dx + 2\int_2^3 dx + 3\int_3^4 dx + \cdots + (n-1)\int_{n-1}^{n} dx$$

$$= (2-1) + 2(3-2) + \cdots + (n-1)(x-n+1)$$

$$= 1 + 2 + 3 + \cdots + (n-1) = \frac{(n-1).(n-1+1)}{2}$$

$$= \frac{n(n-1)}{2}$$

$$\int_0^n \{x\}dx = \int_0^n (x-[x])dx$$

$$= \left[\frac{x^2}{2}\right]_0^n - \frac{n(n-1)}{2}$$

$$= \frac{n^2}{2} - \frac{n(n-1)}{2} = \frac{n}{2}[n-(n-1)] = \frac{n}{2}$$

$$\therefore \text{Expression} = \frac{\frac{n(n-1)}{2}}{\frac{n}{2}} = n-1$$

9. **Solution : (A) less than or equal to $\frac{\pi}{6}$;**

$$0 < x < \frac{1}{2}$$

$$\therefore x^{2n} \le x^2$$

$$\Rightarrow -x^{2n} \ge -x^2$$

$$\Rightarrow 1-x^{2n} \ge 1-x^2$$

$$\Rightarrow \sqrt{1-x^{2n}} \ge \sqrt{1-x^2}$$

$$\Rightarrow \frac{1}{\sqrt{1-x^{2n}}} \le \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^{2n}}} \le \int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}} = \left[\sin^{-1} x\right]_0^{\frac{1}{2}}$$

$$\Rightarrow I \le \frac{\pi}{6}$$

10. Solution : (B) G.P. ;

Using reduction formula we have

$I_1, I_2, I_3, \ldots$ are in G.P.

11. Solution : (B) $-\frac{y^2}{x^2}$;

$$y = \frac{x}{\ln|c| + \ln|x|}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\ln|cx| - x.\frac{1}{x}}{(\ln|c| + \ln|x|)^2} = \frac{\ln|cx| - 1}{(\ln|cx|)^2} = \frac{\frac{x}{y} - 1}{\left(\frac{x}{y}\right)^2}$$

$$\frac{y}{x} = \frac{1}{\ln|cx|} \Rightarrow \ln|xc| = \frac{x}{y}$$

$$= \frac{1}{\frac{x}{y}} - \frac{y^2}{x^2} = \frac{y}{x} - \frac{y^2}{x^2}$$

$$\therefore \phi\left(\frac{x}{y}\right) = -\frac{y^2}{x^2}$$

12. Solution : (C) three distinct values of k;

$y = e^{kx}$

$$\frac{dy}{dx} = e^{kx}.k = ke^{kx} = ky$$

$$\frac{d^2y}{dx^2} = k\frac{dy}{dx} = kky = k^2y$$

$$\left(\frac{d^2y}{dx} + \frac{dy}{dx}\right)\left(\frac{dy}{dx} - y\right)$$

$$= (k^2y + ky)(ky - y)$$

$$= (k^2 + k)y.(k - 1)y$$

$$= k(k + 1)(k - 1)y^2$$

$$y\frac{dy}{dx} = y.ky = ky^2$$

$\therefore$ k (k +1) (k – 1) = k

$\Rightarrow$ k (k^2 – 1 – 1) = 0

$\Rightarrow$ k (k^2 – 2) = 0

$\therefore$ k = 0, $\pm\sqrt{2}$

13. Solution : (D) ; $\frac{d^2y}{dz^2}+4y=0$

$z=\log_e \tan\frac{x}{2}$

$\frac{dz}{dx}=\frac{1}{\tan\frac{x}{2}}.\sin^2\frac{x}{2}.2=\frac{1}{\sin x}$

$\tan\frac{x}{2}=e^z$

$\therefore \sin x=\frac{2e^z}{1+e^{2z}},\ \cos x=\frac{1-e^{2z}}{1+e^{2z}}$

$\cot x=\frac{1}{2}\left(e^{-z}-e^z\right)$

$\Rightarrow \frac{dx}{dz}=\sin x$

$\frac{dy}{dx}=\frac{dy}{dz}\cdot\frac{dz}{dx}=\frac{dy}{dz}\cdot\left(\frac{1+e^{2z}}{2e^z}\right)$

$\frac{d^2y}{dx^2}=\frac{d}{dx}\left\{\frac{dy}{dz}\cdot\frac{1}{2}\left(e^{-z}+e^z\right)\right\}$

$=\frac{1}{2}\frac{d}{dz}\left\{\frac{dy}{dz}\left(e^{-z}+e^z\right)\right\}\cdot\frac{dz}{dx}$

$=\frac{1}{2}\left[\frac{d^2y}{dz^2}\left(e^{-z}+e^z\right)+\frac{dy}{dz}\left(-e^{-z}+e^z\right)\right]\frac{1}{2}\left(e^{-z}+e^z\right)$

$=\frac{1}{4}\frac{d^2y}{dz^2}\left(e^{-z}+e^z\right)^2+\frac{1}{4}\left(e^{2z}-e^{-2z}\right)\cdot\frac{dy}{dz}$

$\therefore$ Given equation becomes

$\frac{d^2y}{dz^2}+4y=0$

14. Solution : (C) f(x) is discontinuous in [–1, 1] but still has the maximum and minimum value ;

$f(0^-)=1$

$f(0^+)=0$

$\therefore$ f (x) is discontinuous at x = 0.

$f'(x)=\begin{cases}1, & -1\le x<0\\ -1, & 0<x<1\end{cases}$

f(x) is increasing in (–1, 0)

f(x) is decreasing in (0, 1)

$\therefore f_{max}=f(0)=1$

$f_{min}=f(1)=-1$

15. Solution : (C) 200 m ;

$$x = 100t - \frac{25}{2}t^2$$

$$\therefore \frac{dx}{dt} = 100 - \frac{25}{2} \times 2t = 100 - 25t = 0 \Rightarrow t = 4$$

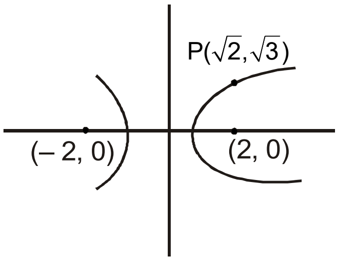
$$\frac{d^2x}{dt^2} = -25 < 0$$

$$\therefore x_{max} = 100 \times 4 - \frac{25}{2} \times 16$$

$= 400 - 200 = 200$ m.

16. Solution : (A) $y = x\sqrt{6} - \sqrt{3}$;

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$2ae = 4$

$ae = 2$

$b^2 = a^2(e^2 - 1)$

$b^2 = 4 - a^2$

$= a^2 + b^2 = 4$

$$\frac{2}{a^2} - \frac{3}{b^2} = 1$$

$\Rightarrow 2b^2 - 3a^2 = a^2b^2$

$\Rightarrow 2(4 - a^2) - 3a^2 = a^2(4 - a^2)$

$\Rightarrow 8 - 2a^2 - 3a^2 = 4a^2 - a^4$

$\Rightarrow a^4 - 9a^2 + 8 = 0$

$\Rightarrow a^4 - a^2 - 8a^2 + 8 = 0$

$\Rightarrow a^2(a^2 - 1) - 8(a^2 - 1) = 0$

$\Rightarrow (a^2 - 1)(a^2 - 8) = 0$

$\Rightarrow a^2 = 1,\ a^2 \neq 8.$ $[\because a^2 = 8 \Rightarrow b^2 = -4]$

$b^2 = 3$

$$\frac{x^2}{1} - \frac{y^2}{3} = 1$$

Tangent at P $(\sqrt{2}, \sqrt{3}) : \sqrt{2}x - \frac{\sqrt{3}y}{3} = 1$

$$\Rightarrow \sqrt{2}x - \frac{y}{\sqrt{3}} = 1$$

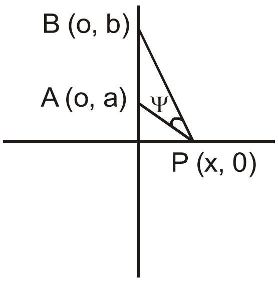
$$\Rightarrow \sqrt{6}x - y = \sqrt{3}$$

$$\Rightarrow y = \sqrt{6}x - \sqrt{3}$$

17. Solution : (A) $x^2 = ab$;

$$m_{AP} = \frac{a-0}{0-x} = -\frac{a}{x}$$

$$m_{BP} = \frac{b-0}{0-x} = -\frac{b}{x}$$



$$\psi = \tan^{-1}\left|\frac{-\frac{a}{x}+\frac{b}{x}}{1+\frac{ab}{x^2}}\right|$$

$$\Rightarrow \psi = \tan^{-1}\left|\frac{b-a}{x}\times\frac{x^2}{x^2+ab}\right|$$

$$= \tan^{-1}\left|\frac{x(b-a)}{x^2+ab}\right|$$

$$\frac{d\Psi}{dx} = \frac{1}{1+\frac{x^2(b-a)^2}{(x^2+ab)^2}}\cdot\frac{(b-a).(x^2+ab)-(b-a)x.2x}{(x+ab)^2} = 0$$

$$\Rightarrow x^2+ab-2x^2 = 0$$

$$\Rightarrow x^2 = ab$$

18. Solution : (No option matching) ;

The problem consists in finding the average value of the function $f(x) = 2y - 2\frac{b}{a}\sqrt{x^2-a^2}$ over the interval [a, 2a]

$$\bar{y} = 2\cdot\frac{1}{a}\int_a^{2a}\frac{b}{a}\sqrt{x^2-a^2}\,dx$$

$$= \frac{2b}{a^2}\left[\frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\,ln\left(x+\sqrt{x^2-a^2}\right)\right]_a^{2a}$$

$$= b\left[2\sqrt{3} - ln\left(2+\sqrt{3}\right)\right]$$

19. Solution : (C) $-\frac{1}{\sqrt{3}}$;

$$V = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1(1-0) - a(0-a^2) + 1(0-a) = 1 + a^3 - a$$

$$\frac{dV}{dx} = 3a^2 - 1 = 0 \Rightarrow a = \pm\frac{1}{\sqrt{3}}$$

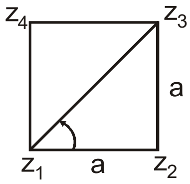
$$\Rightarrow \frac{d^2V}{dx^2} = 6a = 6\times\left(-\frac{1}{\sqrt{3}}\right) < 0$$

$$\therefore a = -\frac{1}{\sqrt{3}}$$

20. Solution : **(D) $-iz_1 + (1 + i)\, z_2$;**

$$\frac{z_3 - z_1}{z_2 - z_1} = \frac{|z_3 - z_1|}{|z_2 - z_1|} e^{i\frac{\pi}{4}}$$

$$= \frac{\sqrt{2}a}{a}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = 1 + i$$



$z_3 - z_1 = (1 + i)\,(z_2 - z_1)$

$\Rightarrow z_3 = z_1 + (1 + i)\,(z_2 - z_1)$

$= (1 - 1 - i)\, z_1 + (1 + i)\, z_2$

$= -\,iz_1 + (1 + i)\, z_2$

21. **Solution : (C) $\frac{r^2(n^2-1)}{12}$;**

$a_1,\ a_2 = a_1 + r,\ a_3 = a_1 + 2r,\ a_4 = a_1 + 3r,\ a_5 = a_1 + 4r,\ \ldots,\ a_n = a_1 + (n - 1)\, r$

$$\therefore \bar{x} = \frac{a_1 + a_2 + a_3 + \cdots + a_n}{n} = \frac{\frac{n}{2}[2a_1 + (n-1).r]}{n} = a_1 + \left(\frac{n-1}{2}\right)r$$

$$\sum x_i^2 = a_1^2 + a_2^2 + a_3^2 + \cdots + a_n^2$$

$$= a_1^2 + (a_1 + r)^2 + (a_1 + 2r)^2 + (a_1 + 3r)^2 + \cdots + \{a_1 + (n-1)r\}$$

$$= na_1^2 + 2a_1r\left(1 + 2 + 3 + \ldots + \overline{n-1}\right) + r^2\left(1^2 + 2^2 + \ldots \overline{n-1}^2\right)$$

$$= na_1^2 + 2a_1r\cdot\frac{(n-1).(n-1+1)}{2} + r^2\cdot\frac{(n-1)(n-1+1)\{2(n-1)+1\}}{6}$$

$$= na_1^2 + a_1r.n(n-1) + \frac{r^2n(n-1)(2n-1)}{6}$$

$$\frac{\sum x_i^2}{n} = a_1^2 + a_1r(n-1) + \frac{r^2(n-1)(2n-1)}{6}$$

$$\therefore \text{Required result} = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$= a_1^2 + a_1 r(n-1) + r^2 \frac{(n-1)(2n-1)}{6} - \left\{a_1 + \left(\frac{n-1}{2}\right)r\right\}^2$$

$$= a_1^2 + a_1 r(n-1) + r^2 \frac{(2n^2-3n+1)}{6} - a_1^2 - a_1 r(n-1) - \frac{(n-1)^2}{4}.r^2$$

$$= r^2\left[\frac{2n^2-3n+1}{6} - \frac{n^2-2n+1}{4}\right]$$

$$= \frac{r^2}{12}(4n^2 - 6n + 2 - 3n^2 + 6n - 3) \quad = \frac{r^2(n^2-1)}{12}$$

22. Solution : (B) ; $1 - \log_3 4$

$$2\log_9\left(3^{1-x}+2\right) = 1 + \log_3\left(4.3^x - 1\right)$$

$$\Rightarrow \log_3\left(3^{1-x}+2\right) = \log_3 3\left(4.3^x - 1\right)$$

$$\Rightarrow 3^{1-x} + 2 = 12.3^x - 3$$

$$\Rightarrow \frac{3}{a} + 2 = 12.a - 3 \text{, where } 3^x = a$$

$$\Rightarrow 3 + 2a = 12a^2 - 3a \Rightarrow 12a^2 - 5a - 3 = 0$$

$$\Rightarrow 12a^2 - 9a + 4a - 3 = 0$$

$$\Rightarrow 3a(4a-3) + 1(4a-3) = 0$$

$$\Rightarrow (4a-3)(3a+1) = 0$$

$$\Rightarrow 4a - 3 = 0,\ 3a + 1 = 0$$

$$\Rightarrow a = \frac{3}{4},\quad a = -\frac{1}{3}$$

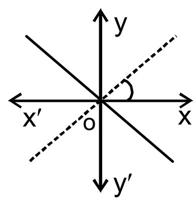
$$\Rightarrow 3^x = \frac{3}{4},\quad 3^x \neq -\frac{1}{3}$$

$$\Rightarrow x = \log_3\frac{3}{4} = 1 - \log_3 4$$

23. Solution : (A) ; $az + \overline{az} = 0$

$$\bar{a}z + a\bar{z} = 0$$

Let a = 2 + 3i



(2 – 3i) (x + iy) + (2 + 3i)(x – iy) = 0

$\Rightarrow$ 2x + i2y – 3xi + 3y + 2x – i2y + i3x + 3y = 0

$\Rightarrow$ 4x + 6y = 0

$\Rightarrow$ 2x + 3y = 0

$$\Rightarrow y = -\frac{2}{3}x$$

Reflection on x-axis is $y = \frac{2}{3}x$

$\Rightarrow 2x - 3y = 0$ .... (i)

(A) $(2 + 3i)(x + iy) + (2 - 3i)(x - iy) = 0$

$\Rightarrow 2x + i\,2y + i\,3x - 3y + 2x - i\,2y - i\,3x - 3y = 0$

$\Rightarrow 4x - 6y = 0$

$\Rightarrow 2x - 3y = 0$ which is identical with (i)

(B) $(2 - 3i)(x + iy) - (2 + 3i)(x - iy) = 0$

$\Rightarrow 2x + i\,2y - i\,3x + 3y - 2x = 0$ is not identical with (i)

(C) $(2 + 3i)(x + iy) - (2 - 3i)(x - iy) = 0$ is not identical with (i)

(D) $\frac{2+3i}{x+iy} + \frac{2-3i}{x-iy} = 0$

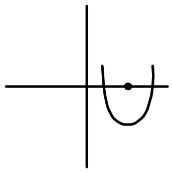
$\Rightarrow (2 + 3i)(x - iy) + (2 - 3i)(x + iy) = 0$

$\Rightarrow 2x - i\,2y + i\,3x + 3y + 2x + i\,2y - i\,2y - i\,3x + 3y = 0$

$\Rightarrow 4x + 6y = 0$ is not identical with (i)

24. Solution : (D) ; $4 + 2p - q^2 < 0$

$x^2 + px - q^2 = 0,\ a > 0$



$f(2) < 0$

$4 + 2p - q^2 < 0$

25. Solution : (D) ; $\frac{8!}{3!}$

EIA

Required number of ways = $\frac{8!}{3!}$

26. Solution : (D) ; $n^n - n!$

Total ways in which atleast one of them will not get any object

= Total ways of distribution – Everyone gets one object.

$= n^n - n!$

27. Solution : (B) ; There exists prime integer which divides P(n)

$P(n) = 3^{2n+1} + 2^{n+2}\ n \in N$

$\boxed{n=1}$ $P(1) = 3^3 + 2^3 = 35$ .... (1)

$\boxed{n=2}$ $P(2) = 3^5 + 2^4 = 243 + 16 = 259$ .... (2)

$\therefore$ From (1) (D) cannot be true

From (2) (C) cannot be true

A is not true

28. Solution : (A) ; $^{2n}C_{n-1}$

For making subset "P"

No of ways of selecting is $^nC_0, {}^nC_1, {}^nC_2 \ldots\ldots {}^nC_{n-1}$

For making subset "Q"

$^nC_1, {}^nC_2, {}^nC_3 \ldots\ldots {}^nC_n$ respectively

$\therefore\ {}^nC_0.{}^nC_1 + {}^nC_1.{}^nC_2 \ldots\ldots {}^nC_n.{}^nC_{n-1}$

$= {}^{2n}C_{n-1}$

29. Solution : (A) ; A + B is singular

For orthogonal matrix

$AB = I_n$

$\therefore\ |A|\,|B| = 1$

$\therefore\ |A|^2 = 1 \quad \therefore\ |A|\ \pm 1$

Let $|A| = 1 \quad \therefore\ |B| = -1$; As $|A| + |B| = 0$ (Given)

$\therefore\ |A + B| = 0$

30. Solution : (A) ; det A is divisible by 11

$$|A| = \begin{vmatrix} 2 & 0 & 3 \\ 4 & 7 & 11 \\ 5 & 4 & 8 \end{vmatrix}$$

$= 2(56 - 44) + 3(16 - 35)$

$= 2(12) - 3(19)$

$= 24 - 57 = -33$

$\therefore\ |A|$ is divisible by 11

31. Solution : (C) ; $(2008)^2$

$$M_r = \begin{pmatrix} r & r-1 \\ r-1 & r \end{pmatrix}, \quad r = 1, 2, 3$$

$$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \det M_1 = 1^2 - 0^2$$

$$M_2 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \Rightarrow \det M_2 = 2^2 - 1^2$$

$$M_3 = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \Rightarrow \det M_3 = 3^2 - 2^2$$

$$M_{2008} = \begin{pmatrix} 2008 & 2007 \\ 2007 & 2008 \end{pmatrix} \Rightarrow \det M_{2008} = (2008)^2 - (2007)^2$$

$\therefore\ \det M_1 + \det M_2 + \ldots\ldots\ldots\ldots + \det M_{2008} = (2008)^2$

32. Solution : (A) ; $b^2 - 4ac$

We have α, β are the roots of $ax^2 + bx + c = 0$. then $\alpha + \beta = -\frac{b}{a}$

$\alpha\beta = \frac{c}{a}$

$$\therefore\ \Delta = \begin{vmatrix} 3 & 1+s_1 & 1+s_2 \\ 1+s_1 & 1+s_2 & 1+s_3 \\ 1+s_2 & 1+s_3 & 1+s_4 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^2 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}^2$$

$$= \left\{\alpha\beta - (\alpha+\beta) + 1\right\}^2 \left\{(\alpha+\beta)^2 - 4\alpha\beta\right\}$$

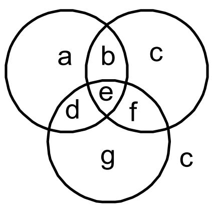
$$= \left(\frac{c}{a} + \frac{b}{a} + 1\right)^2 \left(\frac{b^2}{a^2} - \frac{4c}{a}\right)$$

$$= \frac{(a+b+c)^2(b^2-4ac)}{a^4}$$

$\therefore\ K = b^2 - 4ac$

33. Solution : (B) ; $(A\backslash B)\backslash C = A\backslash(B \cup C)$

Let the regions be marked as follows



Now, (B) $(A\backslash B)\backslash C$ (L.H.S)

= a

$A\backslash(B \cup C)$

= a

34. Solution : (B) ; $\rho_1 \circ \rho_2$ is not transitive relation

Let $\rho_1 = \{(1, 2), (4, 3)\}$

$\rho_2 = \{(1, 4), (3, 1), (3, 4)\}$

$\rho_1 \circ \rho_2 = \{(1, 3), (3, 2), (3, 3)\}$ $(1, 2) \in \rho_1 \circ \rho_2$

Not transitive

35. Solution : (A) ; $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$

$P(A\cap B)=P(A).P(B)=\frac{1}{12}$ (Let P(A) = x, P(B) = y $\therefore xy=\frac{1}{12}$

$P(A'\cap B')=(1-P(A))(1-P(B))=\frac{1}{2}$

$\Rightarrow (1-x)(1-y)=\frac{1}{2}$

$\Rightarrow 1-(x+y)+xy=\frac{1}{2}$

$\Rightarrow x+y=\frac{7}{12}$

$\therefore x+y=\frac{7}{12}$ and $xy=\frac{1}{12}\Rightarrow x=\frac{1}{3}, y=\frac{1}{4}$

$\therefore P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$

36. Solution : (A) ; $p_1 < p_{10} < p_4$

$E_k=\{(a,b)\in S : ab=K\}$

$p_k=p(E_k)$

$\therefore E_1=\{(1,1)\}$ $\quad E_{10}$ = {(2,5), (5 ,2)}, E_4 = {(1,4), (4, 1) (2,2)}

$\therefore p_1=\frac{1}{36},\ p_{10}=\frac{2}{36}, p_4=\frac{3}{36}$

$\Rightarrow p_1 < p_{10} < p_4$

37. Solution : (B) ; $2n\pi\pm\frac{\pi}{3}$

$\frac{1}{6}\sin\theta\tan\theta=\cos^2\theta$

$\Rightarrow \sin^2\theta=6\cos^3\theta$

$\tan^2\theta=6\cos\theta$

$\tan^2\frac{\pi}{3}=6\cos\frac{\pi}{3}$

Check option only B is matching.

38. Solution : (D) ; a pair of straight lines

$$\left(r\cos\left(\theta-\frac{\pi}{3}\right)\right)^2=\left(\sqrt{2}\right)^2$$

$$r\cos\left(\theta-\frac{\pi}{3}\right)=\pm\sqrt{2}$$

$$r\left(\cos\theta\cos\frac{\pi}{3}+\sin\theta\sin\frac{\pi}{3}\right)=\pm\sqrt{2}$$

$$\frac{x}{2}+\frac{\sqrt{3}y}{2}=\pm\sqrt{2}$$

39. Solution : (B) ; a parabola

$$M_{AB}=\frac{4}{-2t}$$

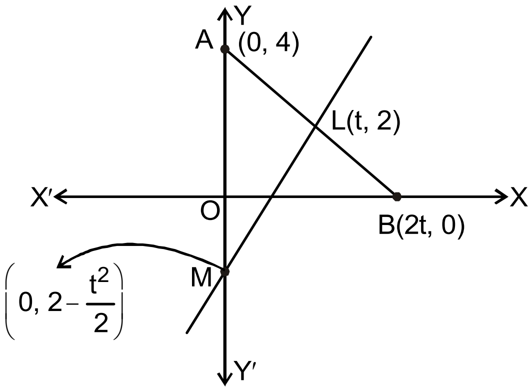
$$M_{AB}=\frac{-2}{t}$$

$$M_{LM}=\frac{t}{2}$$

$$y-2=\frac{t}{2}(x-t)$$

$$y-2=-\frac{t^2}{2}$$

$$y=2-\frac{t^2}{2}\quad\left(\because M=\left(0,2-\frac{t^2}{2}\right)\right)$$



$$h=\frac{t}{2}$$

$$k=2-\frac{t^2}{4}$$

$$\therefore\ \boxed{y=2-x^2}$$

40. Solution : (A) ; (2, 3) or (– 2, –3)

$$4a^2+9b^2+12ab=c^2$$

$\therefore\ c^2 = (2a + 3b)^2$

$c = \pm (2a + 3b)$

$ax + by + c \pm (2a + 3b) = 0$

$ax + by + 2a + 3b = 0$

a(x + 2) + b(y + 3) = 0 $\Rightarrow$ (– 2, – 3)

ax + by – 2a – 3b = 0 $\Rightarrow$ (2, 3)

41. Solution : (D) ; (a, b)

$$\left.\begin{array}{l} x+2y=9 \\ 3x+5y=5 \end{array}\right] \text{Solving tham}$$

3x + 6y = 27

3x + 5y = 5

$\therefore$ $\boxed{y=22}$ x = 9 – 44 $\boxed{x=-35}$

$\therefore ax+by-1=0$ passes through (–35, 22)

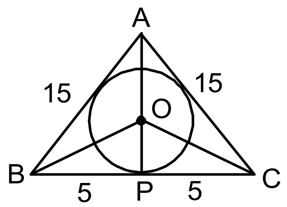
$\therefore$ – 35a + 22b – 1 = 0

35a – 22b + 1= 0

$\therefore$ 35x – 22y + 1 = 0 is given line

So, (a, b)

42. Solution : (B) ; $\frac{5}{\sqrt{2}}$ unit



$$AP=\sqrt{15^2-5^2}=\sqrt{200}=10\sqrt{2}$$

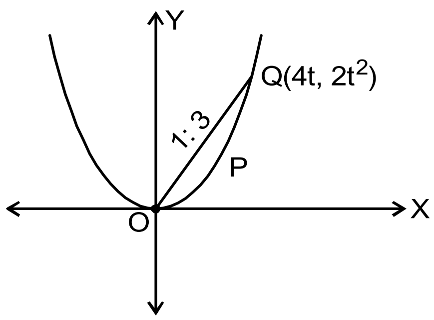
$$\text{Area of } \Delta=\frac{1}{2}\times AP\times BC \;=\frac{1}{2}\times 10\sqrt{2}\times 10$$

$$\Delta=50\sqrt{2}$$

$$S=\frac{15+15+10}{2}=20$$

$$r=\frac{\Delta}{s}=\frac{50\sqrt{2}}{20}=\frac{5}{\sqrt{2}}$$

43. Solution : (D) ; $x^2 = 2y$



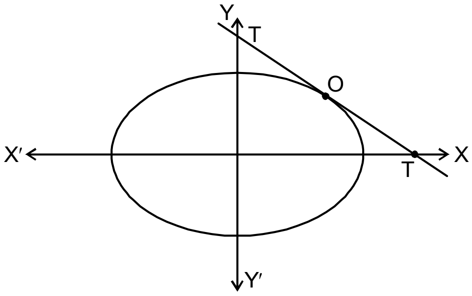
$x^2=8y$ $\therefore$ Parametric point is $\left(4t,2t^2\right)$

$\frac{\not{4}t}{\not{4}} = h$; $\frac{2t^2}{4} = k$

$t^2 = 2k$

$\therefore x^2 = 2y$

44. Solution : (B) ; 2ab



$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$

$$OT \,.\, OT_1 = \frac{a}{\cos\theta} \times \frac{b}{\sin\theta} = \frac{2ab}{\sin 2\theta}$$

45. Solution : (D) ; $-\frac{13}{3}$

$$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$$

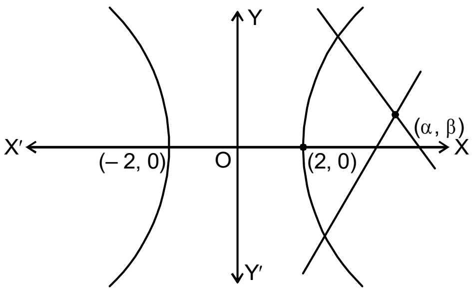
$a = 2$

$b = 3$

$$\frac{a^2x}{a\sec\theta} + \frac{b^2y}{b\tan\theta} = a^2 + b^2$$

$$ax\cos\theta + by\cot\theta = a^2 + b^2$$

$$2x\cos\theta + 3y\cot\theta = 13 \quad \text{... (i)}$$



$$2x\sin\theta + 3y\tan\theta = 13 \quad \text{... (ii) as } \left(\phi = \frac{\pi}{2} - \theta\right)$$

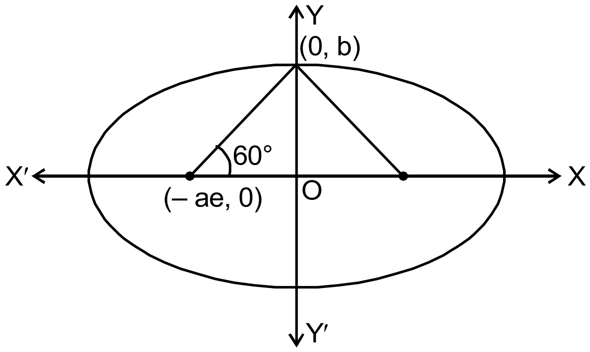
Solving (i) and (ii)

$$3y\cos\theta - 3y\sin\theta = 13(\sin\theta - \cos\theta)$$

$-3y = 13$

$$y = -\frac{13}{3}$$

46. Solution : (B) ; $\frac{1}{2}$



$$\tan 60^\circ = \frac{b}{ae} \Rightarrow e^2 = \frac{b^2}{3a^2} \Rightarrow 3e^2 = \frac{b^2}{a^2}$$

as we know $e^2 = 1 - \frac{b^2}{a^2}$

$\therefore e = \frac{1}{2}$

47. Solution : (C) ; 6

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$\hat{i}(15-16) - \hat{j}(10-12) + \hat{k}(8-9) \Rightarrow -\hat{i} + 2\hat{j} - \hat{k}$ (Normal to the plane)

$\therefore$ Equation of plane is

$(x - 1)(-1) + (y - 2)2 + (z - 3) - 1 = 0$

$x - 2y + z = 0$

$\alpha x - 2y + z - k = 0$. $\because$ Distance possible when planes are parallel.

$\therefore \alpha = 1$

$\therefore$ Distance $= \frac{k}{\sqrt{6}} = \sqrt{6}$

$\therefore \boxed{k = 6}$

48. Solution : (A) ; $\cos^{-1}\left(\frac{2}{3}\right)$

< – 2, – 1, 2> . < 1, 0, 0 >

$\therefore$ Angle $\cos\theta = \frac{2}{3}$

$\therefore \theta = \cos^{-1}\left(\frac{2}{3}\right)$

49. Solution : ⁎, No option matches

Taking $f(x) = x^2 \sin x$,

($\because$ The problem does not mention [.] as G.I.F.)

We get, f(x) is continous everywhere (B)

Where [.] denotes G.I.F,

then, no option matches.

50. Solution : (D) ; $\log^n x$

$y = \log^n x$

Put y = 3, y = ln(ln ln(x))

$$\therefore \frac{dy}{dx} = \frac{1}{\ln(\ln x)} \times \frac{1}{\ln(x)} \times \frac{1}{x}$$

$$\therefore x \log x \log^2 x \log^3 x \frac{dy}{dx}$$

= ln(ln(ln(x))) that is $\log^3 x$

Category – II (Q.51 to 65)

(Carry 2 marks each. Only one option is correct. Negative marks : – ½)

51. Solution : (D) ; 0

$$I = \int_0^{2\pi} \theta \sin^6\theta \cos\theta \, d\theta \qquad \text{.... (1)}$$

$= \theta \to 2\pi - \theta$

$$I = \int_0^{2\pi} (2\pi - \theta)\sin^6\theta \cos\theta \, d\theta \qquad \text{.... (2)}$$

[(1) + (2)] gives

$$2I = 2\pi \int_0^{2\pi} \sin^6\theta \cos\theta \, d\theta$$

$$\Rightarrow I = \pi \left[\frac{(\sin\theta)^7}{7}\right]_0^{2\pi} = 0$$

52. Solution : (C) ; $-k^2$

$x = \sin\theta$, $y = \sin k\theta$

$$\frac{dx}{d\theta} = \cos\theta, \quad \frac{dy}{d\theta} = k\cos k\theta$$

$$y_1 = \frac{dy}{dx} = \frac{k\cos k\theta}{\cos\theta}$$

$y_1\cos\theta = k\cos k\theta \Rightarrow y_1^2\cos^2\theta = k^2\cos^2 k\theta$

$\Rightarrow y_1^2\left(1-\sin^2\theta\right) = k^2\left(1-\sin^2 k\theta\right) \Rightarrow y_1^2\left(1-x^2\right) = k^2\left(1-y^2\right)$

On differentiating,

$\left(1-x^2\right)\left(2y_1y_2\right) + y_1^2(-2x) = k^2(-2yy_1)$

$\Rightarrow (1-x^2)y_2 - xy_1 = -k^2y$

53. Solution : (D) ; changes sign infinitely many times

$f(x) = \sin\left(\frac{1}{x^3}\right)$ is oscillating in nature

54. Solution : (A) ; $\frac{2}{\pi}$

Average ordinate = $\frac{\int_0^{\pi}\sin x\,dx}{\pi - 0} = \frac{2}{\pi}$

55. Solution : (C) ; is constant

$x^{2/3} + y^{2/3} = a^{2/3}$

$x = a\cos^3\theta$

$y = a\sin^3\theta$

$\therefore \frac{dy}{dx} = -\tan\theta = m_T$

$\therefore$ Equation of tangent : $y - a\sin^3\theta = -\tan\theta\,(x - a\cos^3\theta)$

X-axis point $\equiv (a\cos\theta, 0)$, Y-axis point $\equiv (0, a\sin\theta)$

$\therefore$ length of intercept = a unit.

56. Solution : (C) ; 81 cu. unit

$\left(\vec{a}\times\vec{b}\right).\left[\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)\right]$

$= \left(\vec{a}\times\vec{b}\right).\left[\left\{\left(\vec{b}\times\vec{c}\right).\vec{a}\right\}\vec{c} - \left\{\left(\vec{b}\times\vec{c}\right).\vec{c}\right\}\vec{a}\right]$

$= \left(\vec{a}\times\vec{b}\right).\left[\vec{a},\vec{b},\vec{c}\right]\vec{c} - 0 = \left[\vec{a},\vec{b}\,\vec{c}\right]^2$

$\left[\vec{a},\vec{b}\,\vec{c}\right]^2 = 9$

$$\Rightarrow \left[a,\ b,\ \vec{c}\right] = 3\ (\text{unit})^3$$

$$\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)$$

$$= \left\{\left(\vec{a}\times\vec{b}\right).\vec{c}\right\}\vec{b} - \left\{\left(\vec{a}\times\vec{b}\right).\vec{b}\right\}\vec{c} = \left[\vec{a},\ \vec{b},\ \vec{c}\right]\vec{b} - 0 = 3\vec{b}$$

$$\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right) = \left\{\left(\vec{b}\times\vec{c}\right).\vec{a}\right\}\vec{c} - 0 = 3\vec{c}$$

$$\left(\vec{c}\times\vec{a}\right)\times\left(\vec{a}\times\vec{b}\right) = \left\{\left(\vec{c}\times\vec{a}\right).\vec{b}\right\}\vec{a} - 0 = 3\vec{a}$$

$$\therefore \text{ Requried volume } = 3\vec{b}.\left(3\vec{c}\times 3\vec{a}\right)$$

$$= 27.\left[\vec{a},\vec{b},\vec{c}\right] = 27 \times 3 = 81$$

57. Solution : (C) exists at infinitely many points

$f(x) = e^{\sin x} + e^{\cos x}$

$f'(x) = \cos x\, e^{\sin x} - \sin x\, e^{\cos x}$

Now, $\cos x\, e^{\sin x} = \sin x\, e^{\cos x}$

$e^{\sin x - \cos x} = \tan x$

$f'(x) = 0 \quad \therefore$ "C"

Now, f(x) is attaining maxima at many points.

58. Solution : (C) a, b, c cannot be in A.P. or G.P. but can be in H.P.;

$ax^2 + 2bx + c = 0$

Now, no real roots, so $D < 0$

$(2b)^2 - 4ac < 0$

$4b^2 < 4ac$

$b^2 < ac$

Now, a, b, c in G.P $\Rightarrow b^2 = ac$ so, not possible

let a, b, c in A.P and be 1, 2, 3

Now, $2^2 < 3$ contradiction

$\therefore$ Not in A.P

let a, b, c in H.P be $1, \frac{1}{2}, \frac{1}{3}$

$$\left(\frac{1}{2}\right)^2 < \frac{1}{3}$$

$\therefore$ True So, (C) is correct.

59. Solution : (B); n

$$\frac{\frac{a_1}{a_2}+\frac{a_2}{a_3}+\ldots+\frac{a_n}{a_1}}{n} \geq \Rightarrow \sqrt[n]{\frac{a_1}{a_2}\times\frac{a_2}{a_3}\times\ldots\frac{a_n}{a_1}}$$

$$\therefore \frac{a_1}{a_2}+\frac{a_2}{a_3}+\ldots\frac{a_n}{a_1} \geq n$$

$\therefore$ Min value is "n"

60. Solution : (A) ; x = 1 = y

$B = PAP^{-1}$ $\qquad$ $PP^T = P^TP = I$

$\Rightarrow BP = PA$ $\qquad$ $P^{-1} = P^T$

$$\Rightarrow \begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}0&1&0\\x&0&0\\0&0&y\end{bmatrix} = \begin{bmatrix}0&1&0\\x&0&0\\0&0&y\end{bmatrix}\begin{bmatrix}0&0&1\\1&0&0\\0&0&0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x&0&0\\0&0&y\\0&0&0\end{bmatrix} = \begin{bmatrix}1&0&0\\0&0&x\\0&0&0\end{bmatrix}$$

$\Rightarrow x = 1, x = y \Rightarrow x = 1 = y$

*

61. Solution : (A) $A \subset \{x \in \mathbb{N} : 1 \leq x \leq 20\}$ and $B \subset \{y \in \mathbb{N} : 1 \leq y \leq 39\}$;

$2x + y = 41$

$$x = \frac{41-y}{2}$$

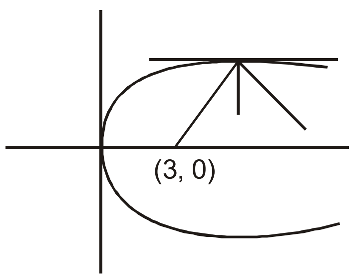
$y = 41 - 2x$

$x = 1, 2, 3 \ldots. 20.$

$1 \leq y \leq 39.$

So (A)

62. Solution : (B) y = 18 ;



All rays moving || to x axis, pass through focus after reflection.

$y^2 = 12\,k$

$$x = \frac{k^2}{12}$$

$$\frac{k-0}{\frac{k^2}{12}-3} = \frac{3}{4}$$

$k^2 - 16k - 36 = 0$

$k^2 - 18k + 2k - 36 = 0$

$k(k - 18) + 2(k - 18) = 0$

$k = -2$

$k = 18$

63. Solution : (C) infinitely many lines with slope ± 1

$\sin^2 x + \sin^2 y = 1$

$\sin^2 x = 1 - \sin^2 y$

$\sin^2 x = \cos^2 y$

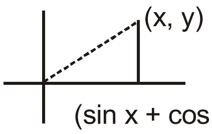
$\sin x = \pm \cos y$

$\sin x = \cos y$

for circle

$\sin x = \cos y$

$\sin y = \sin x$



$(\sin x + \cos y)(\sin x - \cos y) = 0$

$\sin x = \sin\left(\frac{\pi}{2} - y\right)$

$x = n\pi + (-1)^n\left(\frac{\pi}{2} - y\right)$

$\sin x = -\cos y$

$\sin x = \sin y$

So above is a set of infinitely any line with slope ± 1.

64. Solution : (B) $\frac{3}{10}$

$$\left(\frac{1}{2.3} + \frac{1}{2^2 3^2} + ... + \frac{1}{2^n.3^n}\right) + \left(\frac{1}{2^2.3} + \frac{1}{2^3.3^2} + ... + \frac{1}{2^{n+1}.3^n}\right)$$

Both are in G.P.

$$\frac{\frac{1}{2.3}\left(1-\left(\frac{1}{23}\right)^n\right)^0}{1-\frac{1}{2.3}} + \frac{1}{2^2.3}\frac{\left(1-\left(\frac{1}{2.3}\right)^n\right)}{1-\frac{1}{2.3}}$$

$$\frac{\frac{1}{6}}{\frac{5}{6}} + \frac{\frac{1}{12}}{\frac{5}{6}}$$

$$\frac{1}{5} + \frac{1}{12} \times \frac{6}{5} = \frac{1}{5} + \frac{1}{10} \Rightarrow \frac{2+1}{10} = \frac{3}{10}$$

65. Solution : **(A)** ; $y\log y = \tan x \dfrac{dy}{dx}$

$y = e^{a\sin x}$

$\ln y = a \sin x$

$\dfrac{1}{y}\dfrac{dy}{dx} = a\cos x$

$\dfrac{dy}{dx} = ay\cos x = ae^{a\sin x}\cos x$

Category – III (Q.66 to 75)

(Carry 2 marks each. One or more options are correct. No negative marks)

66. **Solution : (B) ;** $3 - \dfrac{\pi}{2}$

$\int_0^x (f'(t) - \sin 2t)dt = \int_x^0 f(t)\tan t\, dt$

on diff.

$f'(x) - \sin 2x = -f(x)\tan x,$ $\qquad f(0) = 1$

$\Rightarrow \dfrac{dy}{dx} + y\tan x = \sin 2x$

I.F. $= e^{\int \tan x dx} = e^{\ln \sec x} = \sec x$

Solution is $y(\sec x) = \int \sin 2x \sec x\, dx \Rightarrow \dfrac{y}{\cos x} = -2\cos x + c$

$\Rightarrow y = -2\cos^2 x + c\cos x$

$f(0) = 1 \Rightarrow c = 3$

$\Rightarrow y = 3\cos x - 2\cos^2 x$

$I = \int_0^{\pi/2} (3\cos x - 1 - \cos 2x)dx$

$= 3[\sin x]_0^{\pi/2} - \left[x + \sin\dfrac{2x}{2}\right]_0^{\pi/2} = 3 - \dfrac{\pi}{2}$

67. **Solution : (B, C) ; H = 112.5 ft; T = 5/2 sec**

$a = 4$ ft/sec^2

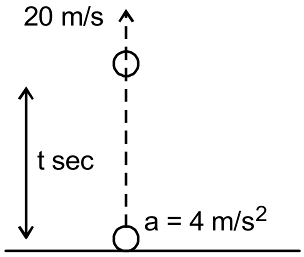
$v = u + at$

$v = 0 + 4 \times 5 = 20$ ft/sec

$v^2 = u^2 + 2as$

$400 = 0 + 2 \times 4 \times s$

S = 50 ft



$$S = ut + \frac{1}{2}at^2$$

$$50 = -20t + \frac{1}{\cancel{2}}\cdot \overset{16}{\cancel{32}}\, t^2$$

$16t^2 - 20t - 50 = 0$

$8t^2 - 10t - 25 = 0$

$$t = \frac{5}{2}$$

$$H = \frac{1}{2}\times 4\times\left(\frac{15}{2}\right)^2 = \frac{225}{2}$$

68. **Solution :** **(B, C) ; f is maximum at two points x = 2 and x = – 1; f is minimum at x = 0**

$$f(x) = 3x^{\frac{2}{3}} - x^2$$

$$f'(x) = \cancel{3}\cdot\frac{2}{\cancel{3}}x^{-\frac{1}{3}} - 2x = 2\left(x^{-\frac{1}{3}} - x\right)$$

$\therefore$ $f'(x) = 0 \Rightarrow 0, -1, 1$

Checking maxima and minima.

69. **Solution :** **(C, D) ; zero; purely imaginary**

$$|z_1 + z_2|^2 = |z_1 - z_2|^2$$

$$(z_1 + z_2)(\overline{z_1} + \overline{z_2}) = (z_1 - z_2)(\overline{z_1} - \overline{z_2})$$

$$\Rightarrow 2(z_1\overline{z_2} + z_2\overline{z_1}) = 0$$

$$\Rightarrow z_1\overline{z_2} = -z_2\overline{z_1}$$

$$\Rightarrow \frac{z_1}{z_2} = -\overline{\left(\frac{z_1}{z_2}\right)}$$

purely imaginary or zero

$\therefore$ $\frac{z_1}{z_2}$ is purely imaginary or zero.

70. **Solution :** **(A, C) ; 482 divides N; N is the product of three distinct prime numbers**

Total number of possible ways = 15 × 15 × 15 = 3375

out of which only one pattern is correct.

Hence correct number of unsuccssful attempt = 3375 – 1 = 3374

N = 2 × 7 × 241

71. Solution : (A, C) ; R^{-1}, $R \cap R^1$

By theorem.

72. Solution : (A, C) ; $(-\infty, -6), (2, \infty)$

$$\frac{x^2}{f(a^2+5a+3)} + \frac{y^2}{f(a+15)} = 1$$

major axis is Y-axis

$\Rightarrow f(a + 15) > f(a^2 + 5a + 3)$

$\Rightarrow a + 15 < a^2 + 5a + 3$

$\Rightarrow a^2 + 4a - 12 > 0$

$\Rightarrow (a + 6)(a - 2) > 0 \Rightarrow$

$a \in (-\infty, -6) \cup (2, \infty)$

73. Solution : None of the option matches

Applying the midpoint of diagonals and slope of sides.

74. Solution : (A, C) ; 0, 2

$f(x) = x^m, m \geq 0$

$f'(x) = m.x^{m-1}$

$f'(a + b) = m(a + b)^{m-1}$

$f'(a) = m(a)^{m-1}$

$f'(b) = m(b)^{m-1}$

$\therefore f'(a + b) = f'(a) + f'(b)$

$\therefore m = 0, 2$

75. Solution : (B, C) ; If f(x) be continuous and periodic with periodicity T, then $I = \int_a^{a+T} f(x)dx$ does not depend on 'a';

Let $f(x) = \begin{cases} 1, & \text{if x is rational} \\ 0, & \text{if x is irrational} \end{cases}$, then f is periodic of the periodicity T only if T is rational.

$I = \int_a^{a+T} = \int_0^T f(x)dx \Rightarrow$ does not depend on 'a'.